Hall effect on multifractality of current distribution of percolation threshold

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LETTER TO THE EDITOR

Hall effect on multifractality of current distribution at percolation threshold

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Abstract. We investigate the scaling behaviour of the current distribution in the random resistor network at the percolation threshold under a weak magnetic field. A self-dual fractal model is used to mimic the two-dimensional percolation cluster at the threshold. The infinite set of exponents is calculated for the moments of the Ohmic and Hall current distribution on the regular fractal. The dependence of the multifractality on the magnetic field by the Hall effect is shown. It is found that the Ohmic and Hall current distribution shows a characteristic multifractality under a magnetic field.

Recently, there has been increasing interest in the critical behaviour of random resistor networks. It has been found that electrical properties of self-similar resistor networks should be characterized by an infinite set of exponents [1–3]. The multifractal structure of the current distribution has been studied [4–7]. Specific members of families of fractal dimensions represent the geometrical and physical substructures of the underlying self-similar structure. Very recently, breakdown of multifractal behaviour in a range of negative moments has also attracted much attention [8–12]. The breakdown phenomenon is due to minimum current decreasing faster than a power law.

The Hall effect has been used extensively to investigate the metal–insulator transition in a variety of disordered systems. An effective medium theory, scaling theories and a simulation approach have been used to discuss the properties of the Hall effect in disordered composite conductors [13–17]. The self-dual fractal model has been proposed to study the scaling properties of the Ohmic and Hall conductivities [18]. The scaling behaviour of the magneto-resistance has also been calculated by using the dual fractal lattice [19]. However, there is an open question as to whether or not the multifractality of the Ohmic and Hall current distribution depends on the magnetic field.

In this letter, we study the dependence of the scaling behaviour of the Ohmic and Hall current distribution on the magnetic field. In order to mimic the infinite cluster at the percolation threshold, we use the self-dual fractal model proposed to study the scaling properties of the Ohmic and Hall conductivities. The self-dual fractal model is a modified variant of the Mandelbrot–Given curve with the self-duality property [18, 20]. Figure 1 shows the generator of the self-dual fractal. The model consists of the two branching Koch curves of which the fractal dimensions of the infinite cluster and its backbone agree with those of the Mandelbrot–Given curve [20]. The Mandelbrot–Given model possesses geometric and topological properties very close to the infinite cluster at the percolation threshold. The dual is represented by the broken line. The two fractal lattices, being self-dual with each other and indicated by the full and broken lines, are electrically unconnected in the absence of a magnetic field H. In the presence

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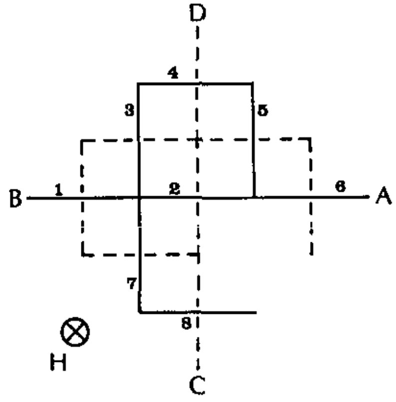

Figure 1. Generator of the self-dual fractal for the infinite cluster. The dual is represented by the broken line. The two lattices indicated by the full and broken lines are electrically unconnected for $H=0$ but are correlated with each other in the presence of a magnetic field H.

of a magnetic field H taken to lie along the z axis, a Hall current will flow through a conductor in the x (or y) direction that depends on its Hall conductance and on the voltage across the y (or x) conductor of the same doublet. Then, the two fractal lattices are correlated with each other by virtue of the unconnected doublets in the presence of a magnetic field H.

We consider the current conservation. In addition to the Ohmic conductance σ_a of each member a of the unit element, there is also a Hall conductance λ_a and Hall coefficient R_a, connected by

$$\lambda_a = \begin{cases} \sigma_a^2 R_a H & \text{for } a \perp H \\ 0 & \text{for } a \parallel H. \end{cases} \tag{1}$$

The current J_a is given by

$$J_a = \sigma_a V_a - \lambda_a V_{a\times H} \tag{2}$$

where $V_{a\times H}$ is the voltage across the dual conductor of the same unit element—the one that is perpendicular to both a and H. Current conservation at the internal point i leads to the following equation for the potentials V_j:

$$\sum_j \sigma_{ij}(V_i - V_j) + \sum_{ij\times H} \lambda_{ij} V_{ij\times H} = 0 \tag{3}$$

where the first summation over j indicates the sum over the nearest-neighbour sites to i, and the second summation over $ij\times H$ represents the sum over the dual conductors of the same unit element as the ij bond. We calculate the Ohmic and Hall currents flowing through each bond on the generator in figure 1. The currents on the bonds (labelled by 1-8) are given by

$$\begin{aligned} j_1 &= \tfrac{4}{11}\sigma(V_B - V_A) - (\tfrac{4}{11})^2\lambda(V_C - V_D) \\ j_2 &= \tfrac{3}{11}\sigma(V_B - V_A) - \tfrac{3}{11}\tfrac{4}{11}\lambda(V_C - V_D) \\ j_3 &= \tfrac{1}{11}\sigma(V_B - V_A) - \tfrac{1}{11}\tfrac{4}{11}\lambda(V_C - V_D) \\ j_4 &= j_3 \qquad j_5 = j_3 \qquad j_6 = j_1 \\ j_7 &= 0 \qquad j_8 = 0 \end{aligned} \tag{4}$$

where we omit the terms of a magnetic field H higher than the first-order term. Similarly, the currents on the dual lattice are calculated. We derive the multifractal exponents of the current distribution by using the currents (4) obtained above. Without generality, we set $(V_B - V_A) = (V_C - V_D) = 1$. Each bond of the generator can be characterized by the fraction of the total Ohmic current flowing through it, i.e. $j' = j/j_{\mathrm{tot\ Ohmic}}$. The moments of the current distribution and corresponding exponent $\zeta(q)$ can be defined by

$$Z(q) \equiv \sum_i j'^q \approx L^{\zeta(q)}. \tag{5}$$

The currents on the fractal lattice are given by a multiplicative process of the currents within the generator. By using equation (4), the exponent $\zeta(q)$ is given by

$$\begin{aligned} \zeta(q) &= \zeta_{\mathrm{Ohmic}}(q) + q \log\{1 - \tfrac{4}{11}(\lambda/\sigma)\}/\log 3 \\ \zeta_{\mathrm{Ohmic}}(q) &= \log\{2 + (\tfrac{3}{4})^q + 3(\tfrac{1}{4})^q\}/\log 3 \end{aligned} \tag{6}$$

where $\zeta_{\mathrm{Ohmic}}(q)$ is the exponent in the absence of a magnetic field H. In the absence of a magnetic field H, the exponents $\zeta_{\mathrm{Ohmic}}(0)$, $\zeta_{\mathrm{Ohmic}}(2)$ and $\zeta_{\mathrm{Ohmic}}(\infty)$ give respectively the exponents $\zeta_{\mathrm{Ohmic}}(0) = \log 6/\log 3 = d_b$ (d_b is the fractal dimension of the backbone), $\zeta_{\mathrm{Ohmic}}(2) = \log \frac{11}{4}/\log 3 = t/\nu$ (t is the exponent of Ohmic conductivity) and $\zeta_{\mathrm{Ohmic}}(\infty) = \log 2/\log 3 = 1/\nu$ (ν is the exponent of correlation length) of backbone, Ohmic conductivity and cutting bonds [18]. In the presence of a magnetic field H, the exponents $\zeta(q)$ change from $\zeta_{\mathrm{Ohmic}}(q)$ to equation (6) except for the exponent $\zeta(0) = d_b$. Figure 2 shows the plot of $\zeta(q)$ against q for various values of λ/σ. The exponents $\zeta(q)$ deviate largely from the $\zeta_{\mathrm{Ohmic}}(q)$ for large $|q|$ or large $|\lambda/\sigma|$. For a negative λ/σ, the exponent $\zeta(q)$ increases with q for $q > 0$. Figure 3 shows the dependence of the typical exponents $\zeta(0)$, $\zeta(2)$ and $\zeta(4)$ on the magnetic field H ($H \approx \lambda$). For a negative value of λ/σ, the line of $\zeta(2)$ intersects with that of $\zeta(4)$, and $\zeta(2)$ is smaller than $\zeta(4)$. The magnetic field has a strong effect on the multifractality of the current distribution.

In summary, we study the scaling behaviour of the current distribution on the self-dual fractal under the magnetic field. We then calculate the multifractality of the

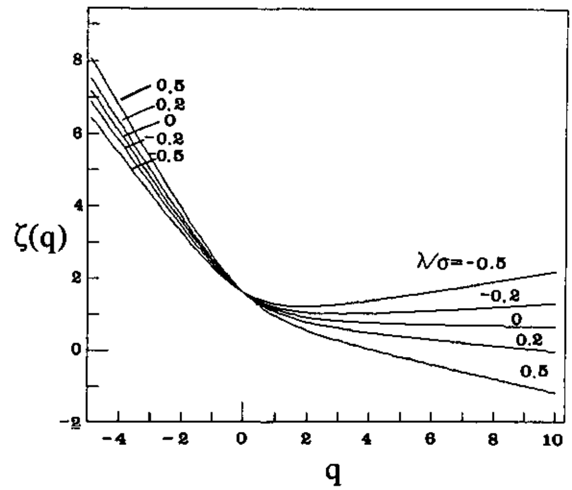

Figure 2. The plots of the exponents $\zeta(q)$ of the moments of the Ohmic and Hall current distribution against q for various values of λ/σ.

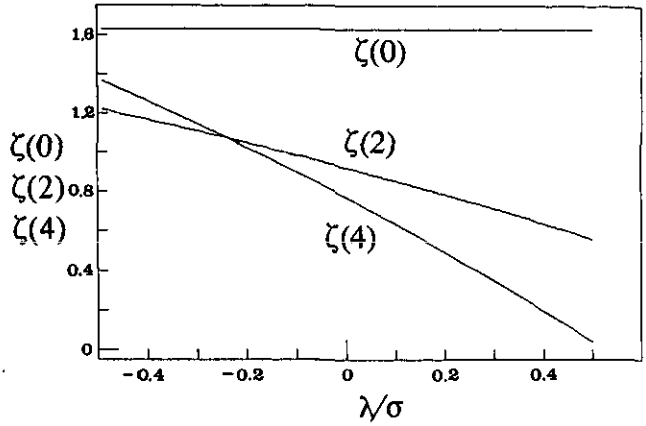

Figure 3. The plots of the typical exponents $\zeta(0)$, $\zeta(2)$ and $\zeta(4)$ against λ/σ ($\lambda \approx H$).

Ohmic and Hall current distribution. We find that the magnetic field has an important effect on the multifractality by the Hall effect.

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